Precise Measurements of the Density arid Critical Phenomena near the Phase Transitions in Helium using High-Q Niobium Microwave Cavities*

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The experimental approach of using high-Q niobium microwave resonator (Q~10⁻) and high resolution thermometry to achieve precise measurements of the density and critical phenomena in liquid helium near phase transitions is described. The numerical verifications of the experimental feasibility as well as the applications of the precision density measurement techniques are discussed

1. INTRODUCTION

The Lambda Point Experiment (LPE) recently conducted in space[1] has demonstrated the importance of microgravity and the high-resolution thermometry (HRT) in advancing fundamental knowledge of continuous phase transitions and in confirming the renormalization group (RG) theory [2]. In this work, we report a new experimental approach which can yield state-of-the art density and critical phenomena measurements near the phase transitions of helium. The principle of this approach is to utilise the HRT technique and the capability of high-resolution and stability in the resonant frequencies of superconducting cavities to detect small changes in the dielectric constant of liquid helium near the lambda transition, thereby providing precision measurement for the specific heat critical exponent α and the amplitude coefficients.

2. THE PHYSICAL CONCEPTS FOR PRECISION DENSITY MEASUREMENTS

The derivation of precise density (ρ) information from the dielectric constant (c) is based on the Clausius-Mossotti relation: $(\varepsilon-1)/(\varepsilon+2) = (4\pi\alpha_0\rho)/(3M)$, where M is the molecular weight, and the polarizability α_0 is assumed to be constant near the lambda transition. In addition, the resonant frequency shift (Δf) of a microwave cavity containing liquid helium is related to the change in the dielectric constant of the liquid helium:

$$\frac{\Delta f}{f_0} , \frac{f - f_0}{f_0} = \frac{\int_{\Omega_0}(\varepsilon - \varepsilon_0)|E|^2 d\Omega}{\int_{\Omega_0} \varepsilon_0|E|^2 d\Omega} \quad (1)$$

where the integration is performed over the entire volume Ω_0 of the microwave cavity, and E is the electric field of the resonant mode.

3. THE EXPERIMENTAL APPROACH

Our experimental techniques involve using high-Q helium microwave resonators (Q~1010) to achieve high frequency resolution (a few parts in 10") for precise measurements of the dielectric constant c in liquid helium near phase transitions. The temperature resolution and stability of ~10⁻¹⁰K are achieved with the use of HRT. The state-of-the-art density measurements of the liquid He near the lambda transition can provide best ever values for the critical exponent α and the amplitude coefficients for the thermal expansion coefficient β_P (under constant pressure P and for a given reduced temperature of $t = (T - T_\lambda)/T_\lambda$):

$$\beta_P = \frac{1}{\rho}\frac{\partial\rho}{\partial T}|_P = A|t|^{-\alpha}(1 + D|t|^\Delta + ...) + B. \quad (2)$$

In addition to obtaining more precise values of α, A, D, A and B, we can investigate the universality of the lambda transition by precise density measurements near $T_\lambda(P)$ for various pressures away from the vapor pressure, as well as near $T_\lambda(X)$ for different ³He concentrations (X) in the ³He-⁴He mixtures. The pressure variations can be achieved by using the height of the liquid helium column in a capacitor level sensor which is connected to a microwave cavity, and the pressure control and readout can be achieved by using the tunnel diode oscillator and the capacitor-inductor circuit developed by C. 'J'. VanDegrift[3]. A precision of one part in 10⁸ can be achieved for the helium height readout, and since it is very well known that $T_\lambda(P)$ shifts downward with the height by $T_\lambda(z) - T_\lambda(0) = -\gamma_0 z$, with $\gamma_0 = 1.273\,\mu K/cm$, the 10 cm column height allows universality to be treated for temperature shifts up to 13 μK. The verification of

universality near $T_\lambda(X)$ for different ^{3}He concentrations can be achieved by fine-tuning X values using the "heat flush" technique[4].

4. NUMERICAL VERIFICATIONS & APPLICATIONS OF THE TECHNIQUE

To test whether the density profile $\rho(z,T)$ can be derived from the measurements of the resonant frequency shift, we consider the microwave cavity at a constant temperature between $T_\lambda(0)$ and $T_\lambda(d)$, where $T_\lambda(d)$, $T_\lambda(0) + Ted$, and d is the height of the cavity. Both the density and the dielectric constant are functions of the position, as shown in Fig.1. We assume that, the expression fitted to the $\rho(T)$ data by Donnelly et al.[5] can be interpolated into the sub-microkelvin region, and we use them to generate the corresponding $\epsilon(T)$ and $[\Delta f(T)/f_c]$, where f_0 denotes the resonant frequency at T, $T_\lambda(0)$. These "experimental values" of $[\Delta f(T)/f_0]$ are used to deconvolute the density profile $\rho(z,T)$, and the results are compared with the original curves fitted to $\rho(T)$ by Donnelly et al. for verification of the feasibility of our experimental approach. The representative results for a microwave cavity of 1.0 cm height, temperature stability of $\delta T/T \sim 10^-$ " and frequency resolution of $\delta f/f ..)0^{13}$, are shown in Figs.2(a) and 2(b) for Earth-bound measurements and for a microgravity environment, respectively. We note that, although our frequency resolution can be up to $\delta f/f .- 10\text{-}16$, the density resolution is limited by the temperature stability $\delta T/T \sim 10^{11}$. As the HRT techniques advance further, higher frequency resolutions will become relevant. In addition, we note that, at the critical exponent α and the amplitude coefficients can be determined to unprecedented precision, approximately to one part in 10^8.

Another application of our technique is the investigation of the gravity-induced superfluid/normal fluid interface. For a given temperature between $T_\lambda(0)$ and $T_\lambda(d)$, the interface thickness $\langle\xi(T)\rangle$ (where the correlation length is given by $\xi(T) = \xi_0|t|^{-\nu}$) is approximately equal to

$$\xi_0 \left[\frac{T_\lambda(0)}{\gamma_0 d}\right] \left\{\left(\frac{T}{T_\lambda(0)} - 1\right)^{1-\nu} + \left(\frac{T_\lambda(d) - T}{T_\lambda(0)}\right)^{1-\nu}\right\}.$$

(3)

Consequently, by measuring the abrupt changes in the $(\Delta f/f_0)$ as the interface enters the microwave cavity from the bottom, we can determine the thickness of the interface regime as a function of the temperature, thereby estimating the critical exponent ν. Since, for a three dimensional system, the scaling relation $3\nu = 2$ holds, the accuracy of the critical exponent α can be independently verified via the measurement of ν.

Finally, our microwave techniques can also be used to study the losses associated with the critical fluctuations by measuring the Q-values of the cavity under the presence of ultrasonic waves on liquid helium. The losses due

to critical fluctuations can provide direct information for the dynamic exponent z[6] near the phase transitions of the ^{3}He-^{4}He mixtures.
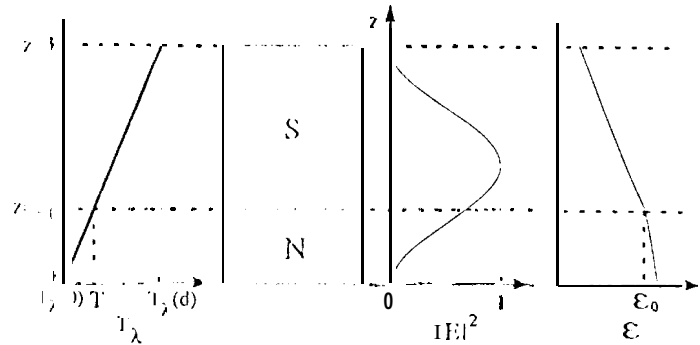

Fig. 1 Schematic illustrations of the gravity-induced variations in $T_\lambda(z)$ and the superfluid (S)/normal fluid (N) interface at $z = z_0$, where $z_0 = [T_\lambda(d) - T_\lambda(0)]/\gamma_0$ is determined by the temperature of the cavity. The profiles of the electric field intensity $|E|^2$ for the TE$_{011}$ mode and of the dielectric constant $\epsilon(z)$ are also shown.
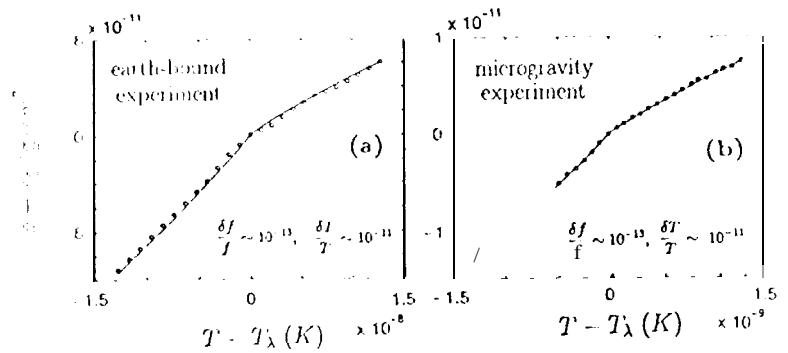

Fig. 2 Comparison of the liquid helium density data obtained from deconvoluting the frequency shifts $(\Delta f/f_0)$ and those from the experimental fittings by Donnelly et al.[5] for (a) Earth-bound measurements and for (b) microgravity environment.

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